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TECHNICAL NOTES

Effect of wall conduction on free convection between asymmetrically heated vertical plates: uniform wall temperature

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INTRODUCTION

THIS NOTE is a companion to ref. [1] wherein the effect of wall conduction on free convection between asymmetrically heated vertical plates is numerically studied. Heating was facilitated by subjecting the external surface of the wall to uniform heat flux (UHF) conditions. In this work consideration is given to the effect of conduction on free convection in an asymmetrically heated channel subjected to uniform wall temperature (UWT) conditions. The geometry, literature review, analysis and solution technique are identical to the one adapted for the UHF case [1]; hence, details of model development and solution technique will not be repeated here.

Only Burch *et al.* [2] studied the impact of wall conduction on laminar free convection between parallel plates. A control-volume based finite difference method was used to solve the governing equations. Only symmetric heating conditions were considered, and external surfaces of the plates were subjected to UWT conditions. A range of geometrical, wall conduction, and heat transfer parameters were addressed. Calculations were made for the Grashof numbers (Gr) range of 10–10^6. This study showed that the wall conduction significantly impacts the heat transfer at high Grashof numbers, low conductivity ratios, and high wall thickness to channel width ratios.

MODEL DEVELOPMENT AND SOLUTION TECHNIQUE

The geometrical configuration for the analysis is very similar to Fig. 1 of ref. [1]. The external surfaces of the left and right walls are heated to UWT conditions. In this work consideration is given to channels subjected to asymmetric heating. The model equations in non-dimensional terms can be written as:

continuity—air

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0; \qquad (1)$$

axial momentum—air

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\frac{\partial^2 U}{\partial Y^2}+\theta; \qquad (2)$$

energy—air

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{Pr}\frac{\partial^2 \theta}{\partial Y^2}; \qquad (3)$$

energy—solid

$$K\left[\frac{\partial^2\theta}{\partial X^2}+\frac{L^2\,Gr^2}{B^2}\frac{\partial^2\theta}{\partial Y^2}\right]=0 \qquad (4)$$

where

$$X=\frac{x}{L\,Gr},\quad Y=y/B$$

$$U=\frac{B^2u}{L\nu\,Gr},\quad V=\frac{Bv}{\nu}$$

$$P=\frac{(p-p_0)B^4}{\rho L^2\nu^2\,Gr^2},\quad Pr=\frac{\mu_a C_{P,a}}{K_a}$$

$$\theta=\frac{T-T_0}{T_{e,l}-T_0},\quad Gr=\frac{g\beta(T_L-T_0)B^4}{L\nu^2}$$

$$\gamma_T=\theta_{e,r}/\theta_{e,l},\quad K=K_s/K_a \qquad (4a)$$

$$K\frac{\partial\theta}{\partial Y}=-\frac{\partial\theta}{\partial Y}\quad \text{at the left interface}$$

$$K\frac{\partial\theta}{\partial Y}=+\frac{\partial\theta}{\partial Y}\quad \text{at the right interface} \qquad (5)$$

$$\theta=1 \text{ (external surface of the left wall)} \qquad (6)$$

$$\theta=\gamma_T \text{ (external surface of the right wall).} \qquad (7)$$

In the above equation, γ_T represents the asymmetric heating parameter.

The above set of model equations along with the boundary conditions were solved using an implicit finite difference scheme, which is discussed in ref. [1]. A series of numerical experiments were conducted to fix the grid size. In the transverse direction, 91 uniform grid points were deployed. Forty grid points were packed in the solid and 51 in the air. In the axial direction, the step size at the channel entrance was made equal to 0.1% of the total length ($L = 1/Gr$) of the channel, and the step size was expanded gradually with an expansion factor of 3%. When the number of grid points was doubled, the variation in total heat transfer rate in the

NOMENCLATURE

B	channel width
Gr	Grashof number
k	thermal conductivity
K	ratio of thermal conductivity of solid wall to the thermal conductivity of air
L	channel height
$\dot{M}$	mass flow rate of air
p	dimensional pressure of air
P	non-dimensional pressure
Pr	Prandtl number
q	non-dimensional heat flux
q'	dimensional heat flux
$\dot{q}$	dimensional heat flow rate
$\dot{Q}$	total heat flow rate
t	wall thickness
T	dimensional temperature
u	dimensional velocity in the axial direction
v	dimensional velocity in the transverse direction
U, V	non-dimensional velocities
x	axial coordinate
y	transverse coordinate
X, Y	non-dimensional coordinates.

Greek symbols

β	thermal expansion coefficient
γ_T	asymmetric heating parameter
θ	non-dimensional temperature
μ	dynamic viscosity of air
ν	kinematic viscosity of air
ρ	density of air.

Subscripts

a	air
e	external surface of the wall
i	interface
l	left wall
max	maximum value
r	right wall
s	solid
0	channel inlet.

channel varied less than 3%. The solution technique was validated by comparing results of the present calculations with those of Burch *et al.* [2]. The non-dimensional mass flow rate of air ($\dot{M}$) through the channel is given by

$$\dot{M} = Gr\frac{L}{B}\int_{Y_{i,l}}^{Y_{i,r}} U\,\mathrm{d}Y \qquad (8)$$

and the non-dimensional heat transfer rate ($\dot{Q}$) through the channel is given by

$$\dot{Q} = \frac{\dot{q}}{Pr\,k_a(T_{e,l}-T_0)} = \frac{L}{B}Gr\int_{Y_{i,l}}^{Y_{i,r}} U\theta\,\mathrm{d}Y. \qquad (9)$$

The calculated values of M and Q are compared with those of existing results in the literature [2, 3] in Table 1. For the case of conducting walls and symmetric heating conditions, the present calculations of M and Q are compared with those of Burch *et al.* [2] in Table 2. The maximum difference between the present calculations and those of Burch *et al.* [2] is less than 3%.

Table 1. Comparison with results for non-conducting walls: $L/B = 0.5$, $t/B = 0$, $\gamma_T = 1$

	$Gr = 10$		$Gr = 10^3$	
	$\dot{M}$	$\dot{Q}$	$\dot{M}$	$\dot{Q}$
Aung *et al.* [3]	0.382	0.535	9.6	7.1988
Burch *et al.* [2]	0.382	0.535	9.34	6.8852
Present work	0.382	0.535	9.377	6.8933

RESULTS AND DISCUSSION

Independent parameters

Careful examination of the governing equations reveals that the independent parameters are the Prandtl number of the fluid (Pr), the Grashof number (Gr), length-to-width ratio of the channel (L/B), ratio of the plate thickness to the channel width (t/B), asymmetric heating parameter (γ_T), and ratio of thermal conductivity of the plate and air (K). The Pr was fixed at 0.7 because only the flow of air between the plates was considered. Calculations were made for $L/B = 1$ and 5 and $t/B = 0$, 0.1 and 0.5. The values of K considered were 1 and 10. The impact of asymmetric heating was studied by performing calculations for $\gamma_T = 0$, 0.5 and 1. The asymmetric heating parameter γ_T is the ratio of the left wall external surface temperature to the right wall external surface temperature, where $\gamma_T = 1$ corresponds to symmetric heating and $\gamma_T = 0.5$ implies that $\theta_{e,l}$ is twice $\theta_{e,r}$. The effect of the buoyancy parameter (Gr) was examined by making calculations for $Gr = 10$–10^4. Higher values of Gr ($>10^4$) required a prohibitively large number of grid points in the axial direction to obtain a grid independent solution.

Table 2. Comparison with results for conducting walls: $\gamma_T = 1$

	Burch *et al.* [2]		Present work	
	$\dot{M}$	$\dot{Q}$	$\dot{M}$	$\dot{Q}$
$Gr = 10$: $t/B = 0.1$, $K = 1$, $L/B = 0.5$	0.368	0.255	0.3725	0.2606
$Gr = 10$: $t/B = 0.5$, $K = 10$, $L/B = 0.5$	0.377	0.268	0.3775	0.2623
$Gr = 10^3$: $t/B = 0.1$, $K = 1$, $L/B = 1$	15.33	4.673	15.772	4.935
$Gr = 10^3$: $t/B = 0.5$, $K = 10$, $L/B = 1$	16.92	5.667	17.144	5.740

Parameters representing wall conduction are K and t/B. The flow properties of interest are transverse velocity distribution (U), axial pressure variation (P), and mass flow rate of air ($\dot{M}$). The heat transfer properties of interest are interface temperature ($\theta_{i,l}$ and $\theta_{i,r}$), interface heat flux ($q_{i,l}$ and $q_{i,r}$), and total heat transfer rate ($\dot{Q}$). Summaries of various parametric runs are given in Tables 3 and 4. Due to space limitations only mass flow and heat transfer rates were chosen for detailed discussion and discussion on other properties such as U, P, $\theta_{i,l}$, $\theta_{i,r}$, $q_{i,l}$ and $q_{i,r}$ can be found in ref. [4].

Mass flow rate and heat transfer rate

Mass flow rate and heat transfer rate are of interest to practising engineers. $\dot{M}$ and $\dot{Q}$ for various parametric runs are shown in Tables 3 and 4. In general, $\dot{M}$ and $\dot{Q}$ increase with increased buoyancy effects. The buoyancy effects depend mainly on the difference between the channel interface temperature and the channel inlet temperature. As Gr and γ_T increase the buoyancy effects increase, and consequently $\dot{M}$ and $\dot{Q}$ increase. From Tables 3 and 4, for a fixed K, t/B and L/B, the mass flow rate ($\dot{M}$) and heat transfer rate ($\dot{Q}$) increase with increase in Gr. With all parameters fixed, $\dot{M}$ and $\dot{Q}$ decrease with increase in t/B because as t/B increases, the wall thermal resistance increases, accordingly, the difference between the interface temperatures and the inlet temperature decreases. This reduction in temperature difference reduces the buoyancy effects, and consequently $\dot{M}$ and $\dot{Q}$ decrease. The effect of increase in K is to reduce the wall thermal resistance which in turn increases interface temperatures. Thus with all parameters fixed, the increase in K increases the buoyancy force, and consequently $\dot{M}$ and $\dot{Q}$ increase. The effect of L/B on $\dot{M}$ and $\dot{Q}$ is apparent from equations (8) and (9). $\dot{M}$ and $\dot{Q}$ are directly proportional to L/B, and as L/B is increased from 1 to 5, $\dot{M}$ and $\dot{Q}$ increase almost by a factor of five as verified in Tables 3 and 4. The physical explanation for this is as L/B increases, the channel width decreases. Decrease in channel width results in

Table 3. Mass flow rate and heat transfer rate for $Gr = 10^4$

γ_T	L/B	t/B	K	$\dot{M}$	$\dot{Q}$	$\theta_{i,l,max}$	$\theta_{i,r,max}$
1	1	0	0	63.873	12.394	1.000	1.000
1	1	0.1	1	49.771	7.319	0.728	0.728
1	1	0.1	10	61.798	11.583	0.963	0.963
1	1	0.5	1	32.983	2.865	0.365	0.365
1	1	0.5	10	55.766	9.198	0.824	0.824
1	5	0.5	1	162.730	14.255	0.375	0.375
0.5	1	0	0	53.600	8.683	1.000	
0.5	1	0.1	1	42.254	5.262	0.729	0.380
0.5	1	0.1	10	51.962	8.149	0.964	0.484
0.5	1	0.5	1	28.233	2.107	0.367	0.199
0.5	1	0.5	10	47.137	6.546	0.826	0.422
0.5	5	0.5	1	139.280	10.488	0.377	0.206
0.5	5	0.1	1	210.450	26.212	0.731	0.380
0.5	5	0.5	10	233.910	32.714	0.842	0.431
0	1	0	0	23.122	5.208		
0	1	0.1	1	19.583	3.289	0.735	2.611×10^{-3}
0	1	0.1	10	22.587	4.918	0.964	2.701×10^{-4}
0	1	0.5	1	15.431	1.363	0.375	5.742×10^{-3}
0	1	0.5	10	21.221	4.018	0.233	7.823×10^{-4}
0	5	0.5	1	75.657	6.802	0.383	8.179×10^{-3}

Table 4. Mass flow rate and heat transfer rate for $Gr = 10^2$

γ_T	L/B	t/B	K	$\dot{M}$	$\dot{Q}$	$\theta_{i,l,max}$	$\theta_{i,r,max}$
1	1	0	0	4.784	3.039		
1	1	0.1	1	4.294	2.561	9.557×10^{-1}	9.557×10^{-1}
1	1	0.1	10	4.727	2.984	9.959×10^{-1}	9.959×10^{-1}
1	1	0.5	1	3.306	1.553	7.460×10^{-1}	7.460×10^{-1}
1	1	0.5	10	4.537	2.764	9.630×10^{-1}	9.630×10^{-1}
1	5	0.5	1	16.220	7.869	0.789	0.789
0.5	1	0	0	3.918	1.961		
0.5	1	0.1	1	3.553	1.689	0.934	0.518
0.5	1	0.1	10	3.876	1.930	0.930	0.503
0.5	1	0.5	1	2.785	1.065	0.709	0.464
0.5	1	0.5	10	3.736	1.803	0.954	0.502
0.5	5	0.5	1	13.662	5.413	0.244	0.498
0.5	5	0.1	1	17.741	8.444	0.936	0.520
0.5	5	0.5	10	18.610	9.078	0.966	0.512
0	1	0	0	2.860	1.072		
0	1	0.1	1	2.640	0.940	0.907	0.075
0	1	0.1	10	2.835	1.057	0.989	0.092
0	1	0.5	1	2.141	0.618	0.662	0.168
0	1	0.5	10	2.752	0.994	0.943	0.036
0	5	0.5	1	10.503	3.158	0.668	0.191

increased flow velocity, leading to increased $\dot{M}$ and $\dot{Q}$. For a practising engineer the maximum temperature along the left and right interfaces is useful information, given in non-dimensional form in Tables 3 and 4. In general, the impact of wall conduction on $\dot{M}$ and $\dot{Q}$ decreases with decrease in γ_T.

CONCLUSIONS

In the numerical study of the effect of wall conduction on laminar heat transfer between two vertical plates subjected to asymmetric heating, walls are heated by subjecting their external surface to constant temperature. The governing equations were solved by an implicit finite difference technique. Calculations were made for a wide range of independent parameters (Gr, t/B, K, L/B and γ_T). The heat transfer and fluid flow in the channel are proportional to the buoyancy forces. Higher values of Gr, K and γ_T contribute to higher buoyancy forces; lower values of t/B result in higher buoyancy forces. The quantitative effect of wall conduction on $\dot{M}$ and $\dot{Q}$ under asymmetric heating conditions can be summarized as follows.

(1) For $Gr = 10^4$, $K = 1$, $t/B = 0.5$, $L/B = 1$ and $\gamma_T = 1$, $\dot{M}$ decreases to 51.6% of the $\dot{M}$ for $t/B = 0$. The heat flow rate reduces to 23.1% of the $\dot{Q}$ for $t/B = 0$. This indicates that the wall conduction reduces $\dot{M}$ and $\dot{Q}$ for $\gamma_T = 1$.

(2) For $Gr = 10^4$, $K = 1$, $t/B = 0.5$, $L/B = 1$ and $\gamma_T = 0$, $\dot{M}$ reduces to 66.7% of the $\dot{M}$ for $t/B = 0$. The $\dot{Q}$ reduces to 26.2% of $\dot{Q}$ for $t/B = 0$. This implies that the asymmetric heating ($\gamma_T = 0$) has less impact on $\dot{M}$ and $\dot{Q}$ than the wall conduction.

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The effect of variable viscosity on convective heat transfer along a vertical surface in a saturated porous medium

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INTRODUCTION

In most of the previous studies on heat transfer in saturated porous media, the thermophysical properties of fluid were assumed to be constant. However, it is known that these properties may change with temperature, especially for fluid viscosity. To accurately predict the heat transfer rate, it is necessary to take into account this variation of viscosity. In spite of its importance in many applications, this effect has received rather little attention.

Previous results [1–4] have shown that when the effects of variable viscosity are taken into consideration, the critical Rayleigh number for the onset of convection is substantially reduced from the classical value, although the associated wave number is nearly the same. For a two-dimensional cavity, it is found that the flow and temperature fields become unstable at even moderate values of the Rayleigh number and exhibit a fluctuating convective state analogous to that observed for the constant viscosity case. In summary, previous studies have considered mostly the instability of the flow and temperature fields caused by the variation of viscosity. Heat transfer results, however, are still very limited. For reported heat transfer data, the working fluids considered are mainly liquids, especially water. For gases, viscosities vary quite differently from liquids. Therefore, it is expected that the heat transfer results for gases also be different from those in liquids. This discrepancy in heat transfer will be further elaborated upon in the following analysis.

In this note, the effect of variable viscosity is considered for mixed convection along a vertical plate embedded in a saturated porous medium. The limiting cases of natural and forced convection are also examined. Similarity solutions are obtained for an isothermally heated plate with fluid viscosity varied as an inverse function of temperature. As pointed out by Cheng and Minkowycz [5], problems of this kind have important applications in geophysics, particularly, geothermal energy extraction and underground storage systems. In addition, it also finds very useful applications in the design of insulation systems employing porous media.

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ANALYSIS

Consider a vertical plate embedded in a saturated porous medium. The fluid and medium properties are assumed to be isotropic and constant, except for the fluid viscosity. The governing equations based on Darcy's law are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u = -\frac{K}{\mu}\left(\frac{\partial p}{\partial x} + \rho g\right) \quad (2)$$